

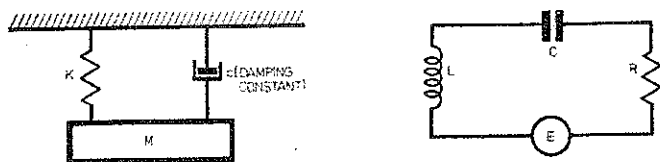
# ANALOGUE COMPUTER

P. J. KRONIS B.Sc.

**PART 4**

- ★ Lissajous Figures
- ★ Flight Simulation
- ★ Special Circuits

It was mentioned last month that damping is caused by air resistance or electrical resistance. This form of damping is called "viscous damping" in mechanical vibrations and is proportional to the velocity. Viscous damping is something that in many cases we wish did not exist. For example, it forces us to burn fuel continuously to propel an aeroplane through the air. Spaceships travelling in vacuum do not have to do this, although they have to burn fuel to decelerate. In other cases we find damping very useful. Mechanical vibration is one of these cases, where damping helps to reduce unwanted and dangerous vibrations. Without it motor cars would provide a very rough ride. The mechanical and electrical damped systems are shown in Fig. 4.1.



**Fig. 4.1. The mechanical and electrical equivalent circuits for damping.**

In the mechanical system damping is provided by a dashpot which is full of viscous oil. This system very closely resembles the suspension unit of a motor car.

The equations describing the response of these systems are:

Mechanical  
 $M\ddot{x} + c\dot{x} + Kx = F$

Electrical  
 $L\ddot{q} + R\dot{q} + 1/Cq = E$

$c$  = damping constant       $q$  = charge, therefore  $\dot{q}$  = current  
 $F$  = a forcing function       $E$  = voltage (function of time)

Table 1 shows the equivalent mechanical and electrical quantities.

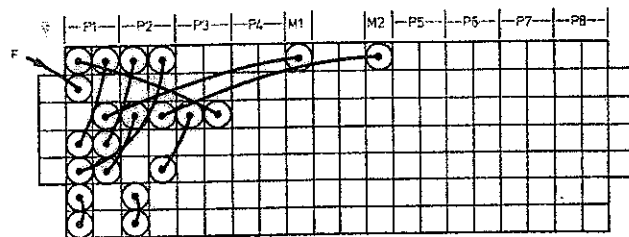
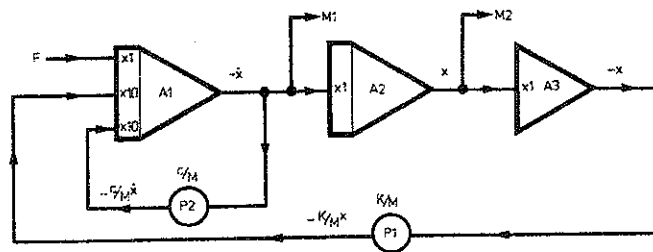
Mechanical		Electrical	
Mass	$M$	Inductance	$L$
Spring stiffness	$K$	1/Capacitance	$1/C$
Force	$F$	Voltage	$E$
Velocity	$\dot{x}$	Current	$\dot{q}$ or $i$
Displacement	$x$	Charge	$q$
Viscous Damping	$c$	Resistance	$R$

**TABLE 1**

Rearranging the mechanical equation we obtain:

$$\ddot{x} = -\frac{c}{M}\dot{x} - \frac{K}{M}x + \frac{F}{M}$$

The flow diagram and the patch panel connections are shown in Fig. 4.2.



**Fig. 4.2. Flow diagram and patch panel layout for investigating a damping system.**

To run the program:

- (1) Check A1 and A2 are integrating, and A3 adding.
- (2) Switch power on and set potentiometer values.
- (3) Computer mode: Hold.
- (4) Press "reset" for a few seconds and release. Switch to compute.
- (5) Apply a forcing function to A1 input.

Several functions can be tried, for example "step", "ramp", "impulse", "sinusoidal", or "random".

### Impulse Function

An impulse function is a force of a relatively large value and short duration. It can be simulated by momentarily applying a voltage to the input of A1. The application of this function produces the results shown in Fig. 4.3. Repeat this experiment with several settings of P2.

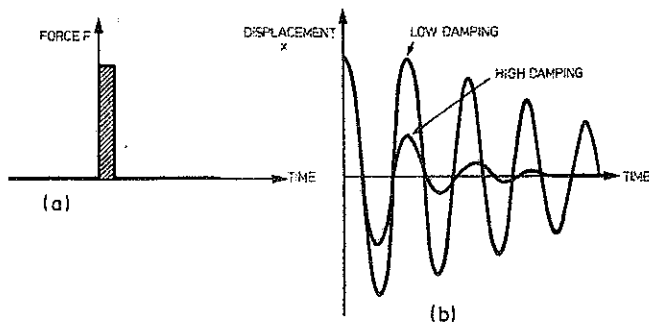


Fig. 4.3. The graphic representation of an impulse function is shown in (a). Graph (b) shows its effect on the circuit in Fig. 4.2.

### Step Function

A step function of about 10V should be applied and a similar procedure followed as with the impulse function. At low damping settings the needle will overshoot the 10V level and oscillate about this value. As more damping is applied the overshoot will become less until at a certain value there will be no overshoot and the needle will take the shortest time to settle at the value of 10V. This is called critical damping. If more damping is applied the needle will take longer to reach the value of 10V. These results are shown in Fig. 4.4.

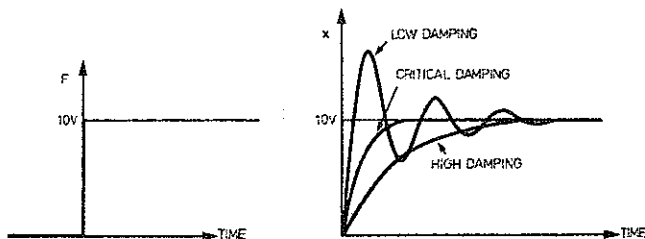


Fig. 4.4. Graph showing the effects of damping after a step function has been applied to Fig. 4.2.

One application of the above is in analogue measuring instruments, like moving coil meters. The input is usually a step function and the needle is damped to a value very near the critical damping value so that the meter can be read with the minimum of delay.

### Sinusoidal and Random Functions

Similar experiments can be carried out by applying sinusoidal and random excitation. We have seen how a sinusoidal function is obtained. A random function is difficult to produce but it can be simulated by adding together two sinusoidal functions of different frequency and amplitude. Three amplifiers are required per sinusoidal function and

therefore nine amplifiers will be needed to experiment with random excitation. One application of this is the problem of the suspension unit of a motor car. Here we have a mass sitting on springs and dampers with the wheels following the uneven contour of the road. The requirement is that as little vibration as possible is transmitted through the suspension unit to the car body. In this case a perfectly satisfactory solution is impossible to obtain and the values of spring stiffness and damping constant are compromises. The best possible values can, however, be worked out using the analogue computer and this saves the designer the expensive and time consuming task of testing many different models of the unit.

### Lissajous Figures

These figures are produced by the combination of two curves, applied at right angles to each other. For example, two sinusoidal functions applied to the X and Y inputs of an X-Y plotter will produce a circle, if they are in phase with each other and have the same frequency. Where the curves are out of phase and differing in frequency, intricate meshing figures are formed, similar to the one shown in Fig. 1.1, at the beginning of this article. In these figures damping was used to vary the amplitude of the oscillation. In real life damping can only be positive, i.e. it can only act to suppress oscillation. However, mathematics, and therefore computers also, allow us to apply negative damping. To do this the flow diagram of Fig. 4.2 is altered by adding an inverting amplifier in the feedback loop containing P2, to change the sign of the damping term in the equation. In this case there is no need to apply a forcing function F. When compute is selected, the oscillation will begin with the amplitude growing progressively until the amplifiers saturate. To produce good figures, very little damping should be applied and the difference in frequency between the X and Y functions should be small.

### Flight simulation

The flight of a rocket can be described by simple mathematical equations involving no more than Newton's law of motion. We can start with a program to simulate the flight of a lunar craft. The diagram with all the forces acting on the spacecraft is shown in Fig. 4.5. The equation of

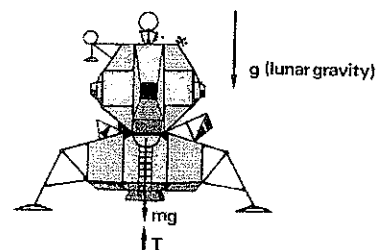


Fig. 4.5. Forces acting on the spacecraft.

motion according to Newton is:

$$T - mg = m\ddot{x}$$

To practice lunar landings, we need to have a thrust control and information about the speed and the altitude of the craft. The flow diagram shown in Fig. 4.6 satisfies these requirements, with potentiometer P1 providing the thrust control and M1 and M2 displaying velocity and altitude respectively. Notice the use of A3 as a sign inverter.

These experiments are of course very simple and the limitations will become very obvious to the programmer. To start with, the equation above assumes that motion can only take place vertically up or down. In reality, movement is in three dimensions and although we can still describe this

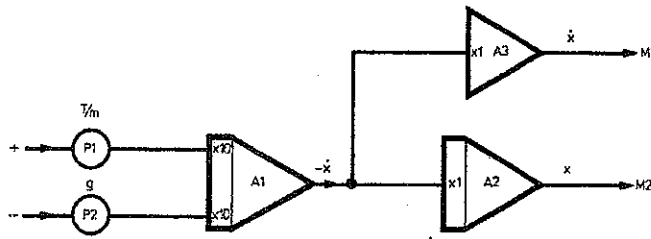


Fig. 4.6. Flow diagram and patch panel layout for amplitude scaling example.

motion mathematically, the equations become more complicated and require more computing elements if it is to be solved.

The other limitation which becomes very apparent when trying to run these programs, is the problem of the limited voltage range over which results can be obtained.

With the equations as they stand, one volt represents 1 unit of the relevant dimension. For example if SI units are used, one volt will represent 1 metre (distance), or 1m/sec (velocity), or 1m/sec<sup>2</sup> (acceleration), or 1 Newton (force). Therefore our flight will be limited to a height of about 26m (if -13m is taken as ground level) and the velocity to ±13m/sec. Suppose that the following data apply to the lunar craft.

- mass  $m = 1,000\text{kg}$ .
- max. engine thrust  $T = 10,000\text{N}$
- lunar gravity  $= 1.6\text{m/sec}^2$ .

Using these values, it will be found that all three amplifiers will saturate very quickly. In fact, with full thrust applied, A1 will saturate in seconds. This is obviously unsatisfactory. Fortunately, we can get round this problem by scaling the various quantities.

### Amplitude scaling

To do this we have to estimate the maximum likely values of the variables and their derivatives. Let us assume for convenience that we may work in the range ±10V and that, in the lunar lander example, we wanted to investigate the flight up to a height of 100m. Hence we may represent 100m by 10V. We therefore apply a scale factor of 10/100 = 0.1. We also estimate that the speed will not exceed 50m/sec. Hence we apply a factor for the velocity of 10/50 = 0.2. So the velocity will be represented by the computer variable 0.2  $\dot{x}$  and the height by 0.1  $x$  both measured in volts. We are also going to scale the acceleration by 0.2 to give a computer variable of 0.2  $\ddot{x}$ . We can now rewrite the equation as:

$$0.2 \ddot{x} = 0.2 (T/m - g)$$

The modified flow diagram in Fig. 4.7 shows the new machine variables in square brackets.

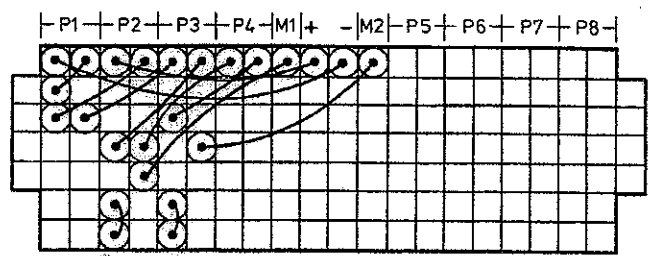
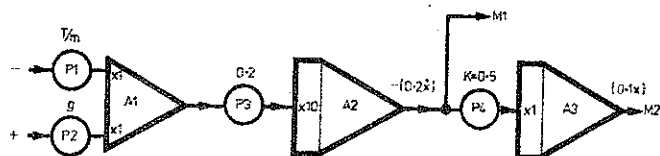


Fig. 4.7. Modified flow diagram and patch panel layout.

To find the value of P4, we have to examine the process of integration carried out by A3.

$$K \int (0.2 \dot{x}) dt = 0.1 x$$

From this we can see that  $K = 0.1/0.2 = 0.5$ . Now if the sensitivity of M1 is adjusted so that it shows 10V at full scale deflection (f.s.d.) then 1 volt will represent 1/0.2 = 5m/sec, and f.s.d. will represent 50m/sec. Similarly 1V on M2 will represent 1/0.1 = 10m. By using P1 as a thrust control, take offs and landings can be practised. To carry out a gentle landing the velocity should be very near zero when the altitude is zero. You should practice doing this with the minimum of hovering, since this is fuel consuming!

The lunar lander can be converted to a vertical take-off aeroplane by the addition of air resistance or drag. The equation and flow diagram is shown in Fig. 4.8.

$$T - mg - D = m\ddot{x}$$

$$\text{or } T - mg - Cd\dot{x}^2 = m\ddot{x}$$

if D is proportional to  $\dot{x}^2$

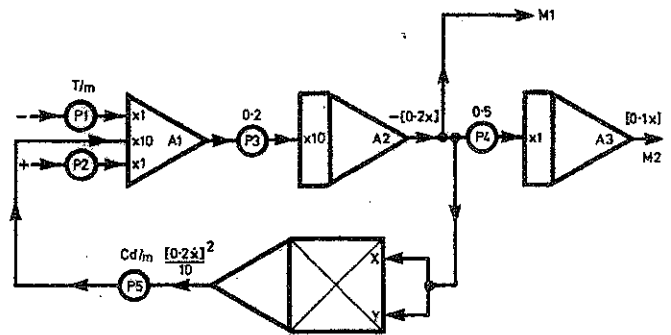


Fig. 4.8. Modified flow diagram to convert the lunar lander to a vertical take-off aeroplane.

### Time Scaling

The solutions to different problems may extend over periods of time ranging from microseconds to many hours. It is not convenient to record results that occur within a split second or results that take many hours to produce. This is not the only reason, however, that we want to avoid both high speed and low speed computer operation. At high speed (e.g. at high frequencies) phase shifts in computing elements and recording instruments produce time lags and therefore errors in the results. At slow speed error voltages tend to build up when integrated. Fortunately we can use time scaling to get round these problems. This we can do by simply reducing or increasing the gain of integrators only. Time is not involved in the operation of adders. Forcing functions should also have their frequencies adjusted by the same factors as the integrators.

To summarize, voltages must be neither too big, nor too small, and the time scale must be neither too fast nor too slow.

### Special Circuits

Many special circuits and techniques are used with analogue computers. Some of these involve the use of diode

shaping circuits to produce non-linear functions. In physical systems we often find such effects as backlash, Coulomb or dry friction, dead space, and we may need to simulate these effects on the analogue computer.

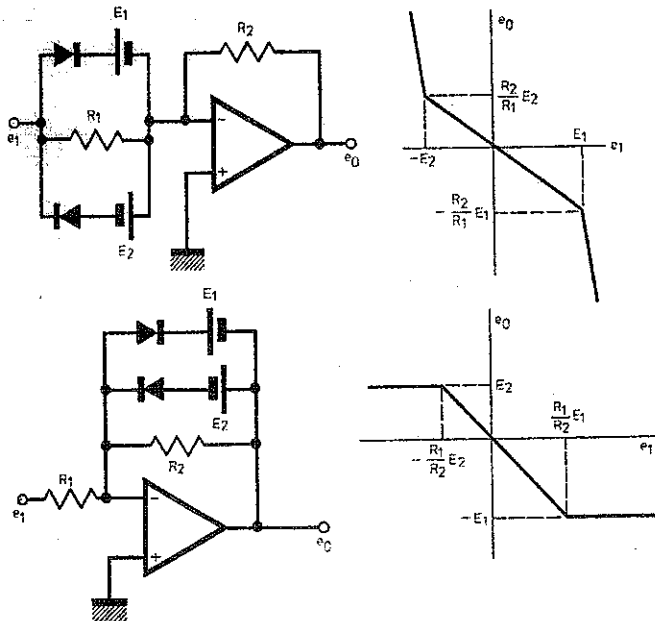


Fig. 4.9. Diode Limiter Circuit.

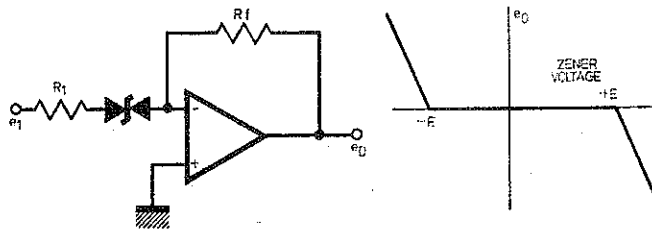


Fig. 4.10. Dead Zone Circuit.

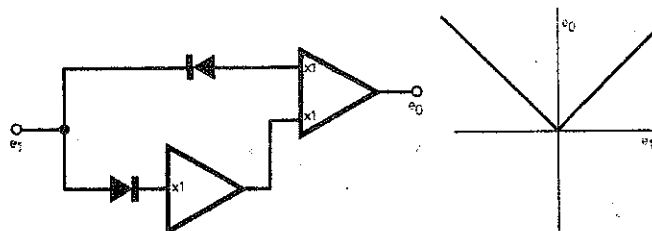


Fig. 4.11. Absolute Value Circuit.

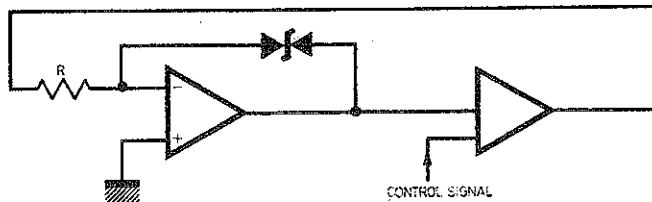


Fig. 4.12. Flip-Flop Circuit.

In this relatively short article, it has not been possible to touch on every subject in the field of analogue computation. Nevertheless it is hoped that the reader has been given a good introduction to the subject and that the constructor will be able to write his own programs and run them on the computer. ★

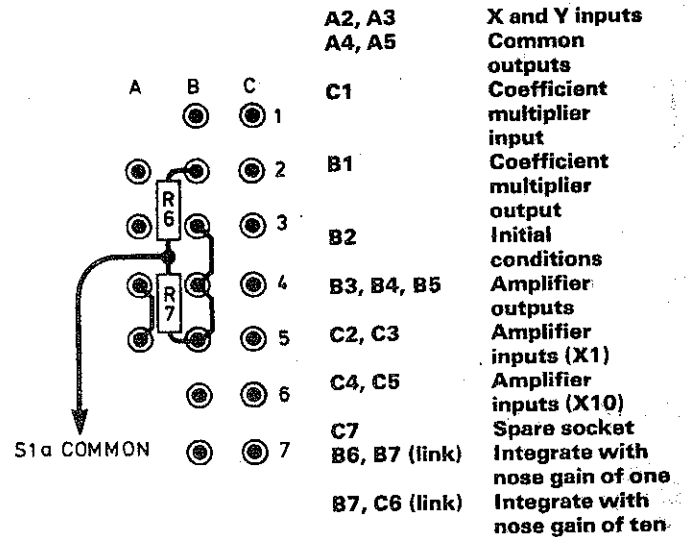


Fig. 3.2. This figure was incorrectly printed in Part 3 of the series.

# News Briefs

## MERSEY MICRO CLUB

A new group, called the Merseyside Mini/Micro Computer Club, has got off the ground under the guidance of Science and Technology Education on Merseyside (STEM). Local distributors have promised to help, in giving talks and demonstrating their products, as well as offering publicity.

Anyone who is interested in microprocessors and small computer systems are very welcome to join, whether new to the world of computing or with many years' experience.

A varied programme is expected to include such subjects as kit building, system configuration, games programs, programming in high level language, business and operating systems.

For further details, contact: Martin Beer (Chairman), Merseyside Mini/Micro Computer Club, 19 Abercromby Square, P.O. Box 147, Liverpool University, Liverpool L69 3BX.

## LATE NIGHT VIEWING

FOLLOWING extensive equipment trials and evaluations, Marconi Avionics Limited (a GEC-Marconi Electronics company) has won an important contract from the Ministry of Defence Procurement Executive for the development of a Visual Augmentation System for RAF Tornado aircraft. The system, which is to be fitted to the Tornado Air Defence Variant, will enable the crew to see a television picture of the scene ahead, taken with a highly-sensitive low light camera system.

The Visual Augmentation System is being developed by the company's Electro-Optical Surveillance Division at Basildon, England. It will show details, to both the pilot and the navigator, of targets at ranges well in excess of those which can be achieved with the unaided human eye and at ambient light levels ranging from daylight to starlight.

The new airborne system is an extension of the company's highly successful low light television systems which are widely used for industrial, commercial and naval applications including surveillance, target tracking and weapon aiming.